

Quadratic Formula

Name _____

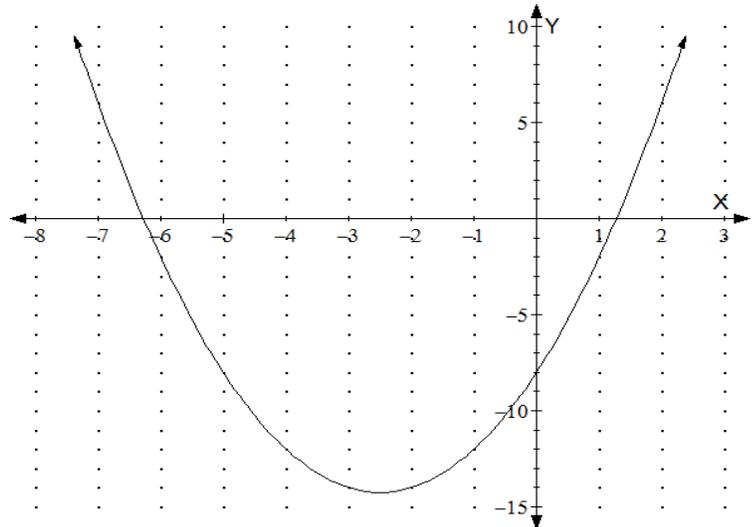
Background: So far in this course we have solved quadratic equations by the square root method and the factoring method. Each of these methods has its strengths and limitations. The quadratic formula allows us to solve any quadratic equation for both the real and complex roots. The derivation of the quadratic formula uses the completing square technique applied to a general quadratic equation of the form $ax^2 + bx + c = 0$.

Quadratic Formula: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. The graph of $y = x^2 + 5x - 8$ is shown below.

a) Fill in the table and plot the points on the graph.

x	y = f(x)
-7	
-3	
-1	
1	
2	
3	



a) What is the y-intercept of the graph?

b) What are the x-intercepts of the graph or the real roots of $f(x)$? Round to 4 decimal places.

c) What value of x does the graph reach its minimum value and what is the minimum value?

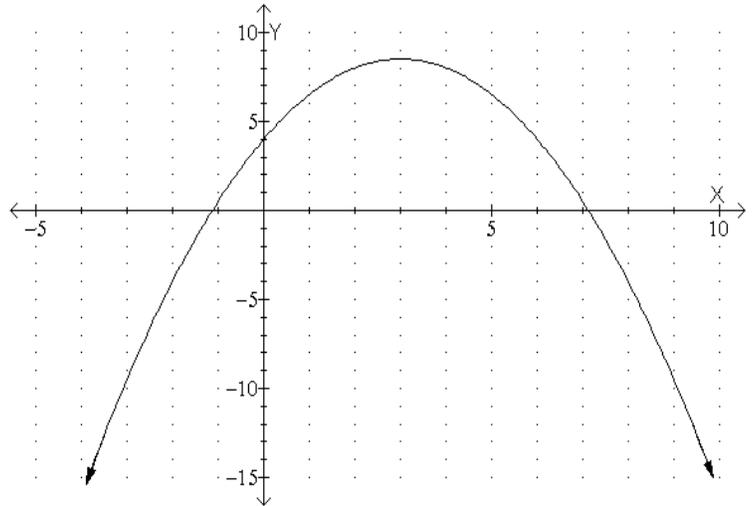
$x_{\min} = \underline{\hspace{2cm}}$ $y_{\min} = \underline{\hspace{2cm}}$

d) What values of x make $y = 8$? (If $f(x) = 8$, what are the values of x ?)

2. The graph of $y = -0.5x^2 + 3x + 4$ is shown below.

a) Fill in the table and plot the points on the graph.

x	y = f(x)
-3	
-2	
-1	
1	
2	
5	



b) What is the y-intercept of the graph?

c) What are the x-intercepts of the graph or the real roots of $f(x)$? Round to 4 decimal places.

d) What value of x does the graph reach its maximum value and what is the maximum value?

$x_{\max} = \underline{\hspace{2cm}}$ $y_{\max} = \underline{\hspace{2cm}}$

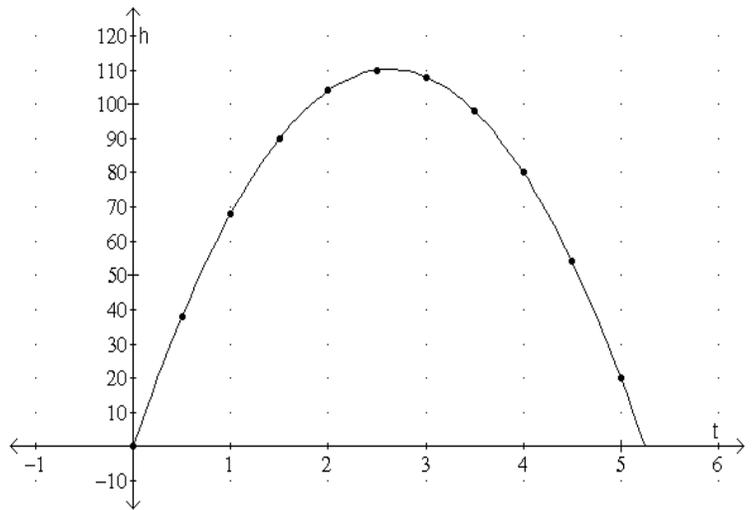
e) What values of x makes $y = 7$?

f) What values of x make $y = -8$?

3. The height above ground of a ball thrown upward with an initial vertical velocity of 84 ft/ sec is described by the equation $h = -16t^2 + 84t$ where h = height in feet above ground at time t and t = time in seconds.

84 ft/sec = 57.3 mph .

- a) How high above ground is the ball when $t = 1$ second ? _____
- b) How high above ground is the ball when $t = 2$ seconds ? _____
- c) What was the average vertical speed of the ball over the time interval [1 sec, 2 sec] ? _____
- d) How high above ground is the ball when $t = 4$ second ? _____
- e) How high above ground is the ball when $t = 4.5$ seconds ? _____
- f) What was the average vertical speed of the ball over the time interval [4 sec, 4.5 sec] ? _____
- g) At what time t did the ball hit the ground ? _____



- h) At what time t did the ball reach its maximum height? $t_{\max} =$ _____
- i) What was the maximum height of the ball above ground? $h_{\max} =$ _____
- j) At what times t was the ball exactly 100 ft above ground? Round to 4 decimal places.

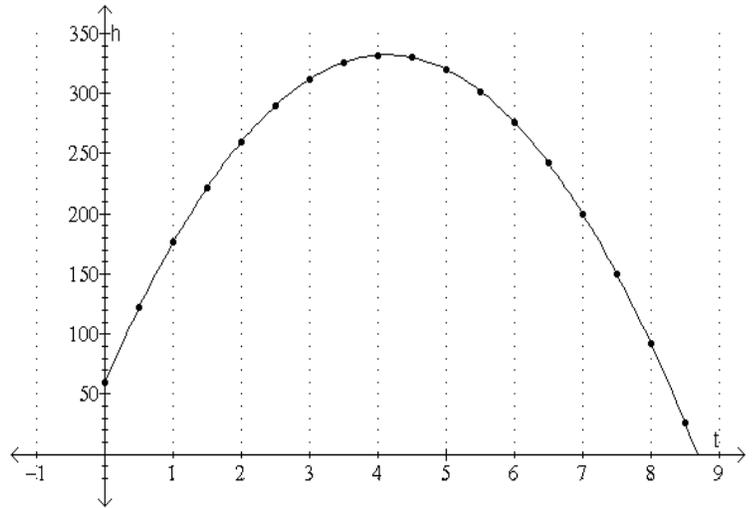
4. From the top of a building, a toy rocket is shot upward with an initial vertical velocity of 132 ft/sec . The formula below gives the height of the rocket above ground in feet at time t . Assume that $t = 0$ when the rocket left its launch pad. Round all answers to nearest 0.0001.

$h = -16t^2 + 132t + 60$ where h = height in feet and t = time in seconds.

132 ft/sec = 90 mph .

- a) How tall is the building?
- b) How high above ground was the rocket when $t = 3$ seconds?
- c) How high above ground was the rocket when $t = 8$ seconds?
- d) At what time t did the rocket hit the ground ?

(The negative solution will be used to find t_{\max} .)



- e) At what time t did the rocket reach its maximum height h ?

$t_{\max} =$ _____

- f) What was the maximum height reached by the rocket?

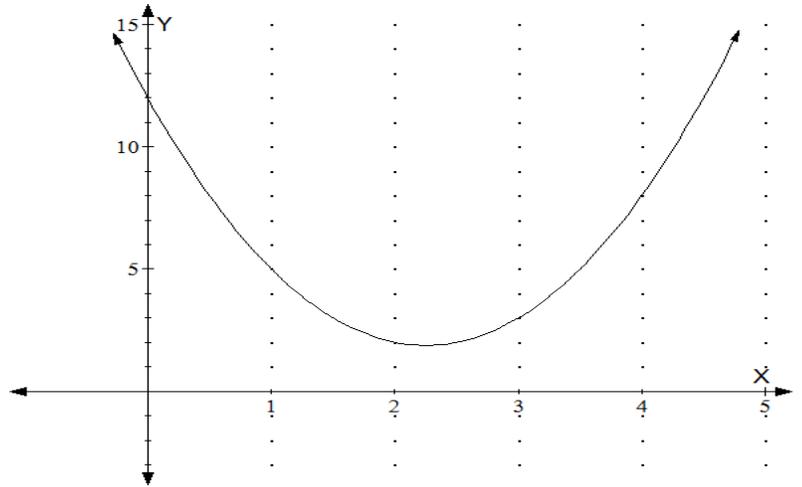
$h_{\max} =$ _____

- g) At what times t was the rocket exactly 300 ft above ground? Round to 4 decimal places.

5. The graph of $f(x) = y = 2x^2 - 9x + 12$ is shown below.

a) Fill in the table and plot the points on the graph.

x	y = f(x)
0	
1	
2	
3	
4	
5	



b) The function $y = f(x)$ does not have any real roots. Explain why this is the case.

c) Use the quadratic formula to find the complex roots of $f(x)$. Write both roots in standard $a + bi$ format.

d) Find the x-y coordinates of the relative minimum point of $f(x)$. Round to 4 decimal places.

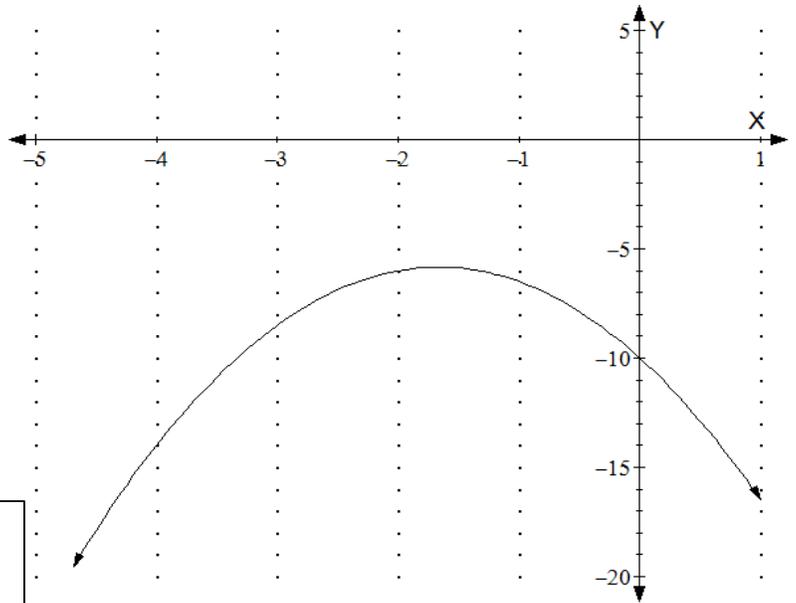
$x_{\min} = \underline{\hspace{2cm}}$ and $y_{\min} = \underline{\hspace{2cm}}$

e) If the roots of a function are real numbers, we can plot the roots on the x-axis where the graph of $f(x)$ intersects the x-axis. Because the roots of $f(x)$ are not complex numbers, we can not plot the roots on the x-axis. If we image the complex number plane layered over the x-y coordinate plane, we can plot the complex roots of $f(x)$. Plot and neatly label the complex roots of $f(x)$ on the x-y coordinate axes above.

6. The graph of $f(x) = y = -1.5x^2 - 5x - 10$ is shown below.

a) Fill in the table and plot the points on the graph.

x	y = f(x)
-5	
-4	
-3	
-2	
-1	
0	



b) The function $y = f(x)$ does not have any real roots. Explain why this is the case.

c) Use the quadratic formula to find the complex roots of $f(x)$. Write both roots in standard $a + bi$ format.

d) Find the x-y coordinates of the relative maximum point of $f(x)$. Round to four decimal places.

$x_{\max} = \underline{\hspace{2cm}}$ and $y_{\max} = \underline{\hspace{2cm}}$

e) If the roots of a function are real numbers, we can plot the roots on the x-axis where the graph of $f(x)$ intersects the x-axis. Because the roots of $f(x)$ are complex numbers, we can not plot the roots on the x-axis. If we image the complex number plane layered over the x-y coordinate plane, we can plot the complex roots of $f(x)$. Plot and neatly label the complex roots of $f(x)$ on the x-y coordinate axes above.