Discrete Probability Distributions - Key

1. Grandma Smith loves to bake cookies for her six precocious grandchildren who are interested in the Poisson probabilities of obtaining various numbers of chocolate chips in one of her cookies. Grandma bakes cookies in batches of 100 cookies and uses 250 chocolate chips per batch. Although Grandma mixes the cookie dough as best she can, the chocolate chips tend to have a random Poisson distribution in the cookie dough and therefore her cookies have various numbers of chocolate chips. Round \( \mu, \sigma \) and \( \lambda \) to four significant decimal places.

a) Describe the interval for this PD.  One of Grandma's cookies  

b) Value of \( \lambda = \frac{2.5 \text{ chips/cookie}}{} \)

c) What constitutes a success?  One success equals finding a chocolate chip in a cookie.

d) Mean of this PD = \( \mu = \frac{2.5}{\text{}} \)  
e) The standard deviation of this PD = \( \sigma = 1.5811 \)

f) Let \( x \) = the number of chocolate chips in one of Grandma's cookies. Find each of the probabilities below. Round to three significant decimal places.

\[
\begin{align*}
P(x = 1) &= 0.205 \\
P(x \leq 3) &= 0.758 \\
P(x > 2) &= P(x \geq 3) = 0.456
\end{align*}
\]

g) What number of chocolate chips in a cookie would be considered unusually high?  
6 or more chips

h) What formula is your graphing using to calculate the probabilities in part (f) above?  
\[
P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}
\]

(i) The grandchildren think that Grandma is too stingy with the chocolate chips, so Grandma started baking batches of four dozen cookies with 300 chocolate chips per batch. Find \( \mu \) and \( \sigma \) for the PD that gives the probability of finding various numbers of chocolate chips in one of Grandma's improved cookies.

\[
\begin{align*}
\mu &= \frac{6.25}{\text{}} \\
\sigma &= \frac{2.5}{\text{}}
\end{align*}
\]

What number of chocolate chips in one of Grandma's improved cookies would be considered unusually low?  
1 or less chips

2. Pyramid Lake is located on the Paiute Indian Reservation in Nevada. The Paiute Nation uses highly trained fish biologists to study and maintain this famous fishery. For the month of November, the catch rate is estimated to be 0.6667 fish per hour for a boat fisherman. Suppose you decide to fish Pyramid Lake for 7 hours during the month of November. Round \( \mu, \sigma \) and \( \lambda \) to four significant decimal places.

a) Describe the interval for this PD.  7 hour time interval in November  

b) Value of \( \lambda = \frac{4.6669}{\text{}} \)

c) Mean of this PD = \( \mu = \frac{4.6669}{\text{}} \)  
d) The standard deviation of this PD = \( \sigma = 2.1603 \)

e) Let \( r \) = the number of fish caught in 7 hours. Find each of the probabilities below. Round to three significant decimal places.

\[
\begin{align*}
P(r = 1) &= 0.0439 \\
P(r = 4) &= 0.186 \\
P(r < 4) &= 0.315 \\
P(r \geq 4) &= 0.685
\end{align*}
\]

f) What formula is your graphing using to calculate the probabilities in part (e) above?  
\[
P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}
\]

(Be specific for this problem.)

g) What number of fish caught in 7 hours would be considered unusually high?  
9 or more fish

h) What number of fish caught in 7 hours would be considered unusually low?  
Zero fish
3. Hairline cracks often appear in cement structures and normally there is nothing to worry about. Cement retaining walls are considered safe if the number of hairline cracks in the wall are spread out and not too close together. Suppose a cement retaining wall is considered safe if the hairline cracks are evenly spread out and cracks appear on average at the rate of 4.2 hairline cracks per 30-foot section of wall. Let us consider the probabilities of finding various numbers of hairline cracks in a 50-foot section of a safe cement retaining wall. Round μ, σ and λ to four significant decimal places.

a) Describe the interval of this PD. 50-foot section of cement retaining wall  

b) Value of λ = 7

c) Mean of this PD = μ = 7           d) The standard deviation of this PD = σ = 2.6458

e) Let x = the number of hairline cracks found in 50-foot section of a safe retaining wall. Find each of the following probabilities. Round to three significant decimal places.

\[ P(x = 3) = 0.0521 \hspace{1cm} P(x \leq 6) = 0.4497 \hspace{1cm} P(x > 6) = 0.5503 \hspace{1cm} P(x \geq 10) = 0.170 \]

f) What number of hairline cracks in a safe 50-foot section of wall would be unusually high? 13 or more cracks

g) What number of hairline cracks in a safe 50-foot section of wall would be unusually low? 1 or less cracks

\[ P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \]

h) Write the formula that your calculator used to find the probabilities in part (e) above. (Be specific for this problem.)

4. Jim is a real estate agent who sells large commercial buildings. Since his commission on the sale of a building is large, he does not need to sell many buildings in order to make a very good living. History shows that Jim sells, on average, 6.8 building in 275 days. Jim would like to know his chances of selling various numbers of buildings in a 60-day period. Round μ, σ and λ to four significant decimal places

a) Describe the interval for this PD. 60-days of commercial building sales  

b) Value of λ = 1.4836

c) Mean of this PD = μ = 1.4836           d) The standard deviation of this PD = σ = 1.2180

f) Let x = the number of commercial buildings Jim sells in a 60-day period. Find each of the following probabilities Round to three significant decimal places.

\[ P(x = 0) = 0.227 \hspace{1cm} P(x \leq 2) = 0.813 \hspace{1cm} P(x > 2) = 0.187 \hspace{1cm} P(x \geq 4) = 0.0636 \]

g) What number of commercial buildings sold in a 60-day period would be unusually high? 4 or more

h) Find λ, μ, and σ for a 180-day selling period.

\[ \lambda = 4.4509, \hspace{1cm} \mu = 4.4509, \hspace{1cm} \text{and} \hspace{1cm} \sigma = 2.1097 \]

What number of commercial buildings sold in a 180-day period would be unusually high? 9 or more

What number of commercial buildings sold in a 180-day period would be unusually low? zero

\[ P(x) = \frac{e^{-4.4509} \cdot 4.4509^x}{x!} \]

Write the formula that your calculator used to find the probabilities for a 180-day period. (Be specific for this problem.)
5. *Consumer Reports* rated airlines and found that 80% of flights arrive on time (that is, within 15 minutes of the scheduled arrival time). Assume that the 80% on time arrival rate is accurate and consider a random sample of 200 flights.

a) This is a binomial pd with 200 Bernoulli trials and probability of success = 0.8

b) What is a success? Flight is on time What is a failure? Flight is not on time

c) Mean of this PD = \( \mu = 160 \)
d) The standard deviation of this PD = \( \sigma = 5.6569 \)
e) On average, how many of the flights in the sample would you expect to arrive on time? 160

f) Let \( x \) = the number of flights in the sample that arrived on time. Find each of the probabilities below. Round to three significant decimal places.

\[
P(x = 160) = 0.0704, \quad P(x \leq 155) = 0.211, \quad P(x < 165) = 0.785, \quad P(x \geq 170) = 0.0430
\]

P(\(x < 165\)) = __________, P(\(x \geq 170\)) = __________, P(\(x > 170\)) = __________

g) Refer to part (f) above. Describe all possible values of \( x \)

\(x = 0, 1, 2, 3, \ldots, 200\)

h) What number of on time arrivals in the sample of 200 flights would be unusually high? 172 or more flights

i) What number of on time arrivals in the sample of 200 flights would be unusually low? 148 or less flights

j) Write the formula that your calculator used to find the probabilities in part (f) above.

\[
P(x) = \binom{200}{x} \cdot 0.8^x \cdot 0.2^{200-x}
\]

6. State Farm Insurance studies show that in Colorado, 55% of the auto insurance claims submitted for property damage were submitted by males under 25 years of age. Consider a random sample of 40 auto insurance claims submitted to State Farm in Colorado.

a) This is a binomial pd with 40 Bernoulli trials and probability of success = 0.55

b) What is a success? An auto insurance claim submitted by a male under 25 years old.

c) What is a failure? An auto insurance claim not submitted by a male under 25 years old.

d) Mean of this PD = \( \mu = 22 \)
e) The standard deviation of this PD = \( \sigma = 3.15 \)

f) On average, how many of the claims would you expect to be filed by a male under 25 years of age? 22

P(\(x = 22\)) = __________, P(\(x \leq 22\)) = __________, P(\(x < 15\)) = __________, P(\(x \geq 30\)) = __________

P(\(x < 15\)) = __________, P(\(x > 29\)) = __________

h) What number of claims in the sample that were filed by a male under age 25 would be unusually high? __________

i) What number of claims in the sample that were filed by a male under age 25 would be unusually low? __________

j) Write the formula that your calculator used to find the probabilities in part (g) above.

\[
P(x) = \binom{40}{x} \cdot 0.55^x \cdot 0.45^{40-x}
\]

(To be specific for this problem.)
7. *USA Today* reported that 25% of all prison parolees become repeat offenders. Alice is a social worker who is currently counseling 27 parolees and is interested in the number of these parolees who do not become repeat offenders. Assume that *USA Today*’s 25% repeat offender rate is accurate.

a) This is a binomial pd with 27 Bernoulli trials and probability of success = ___________.

b) What is a success? A parolee who does not become a repeat offender.

c) What is a failure? A parolee who becomes a repeat offender.

d) Mean of this PD = $\mu = 20.25$

e) The standard deviation of this PD = $\sigma = 2.25$

f) On average, how many of the parolees that Alice is counseling will not become a repeat offender? 20.25

g) Let $x$ = the number of parolees who do not become a repeat offender. Find each of the probabilities below. Round to three significant decimal places.

$$P(x = 20) = 0.172, \quad P(x \leq 18) = 0.214, \quad P(x < 15) = 0.00778, \quad P(x \geq 22) = 0.299$$

h) What number of these parolees who do not become a repeat offender would be unusually high? 25 or more

i) What number of these parolees who do not become a repeat offender would be unusually low? 15 or less

j) Write the formula that your calculator used to find the probabilities in part (g) above. $P(x) = \binom{27}{x} 0.75^x * 0.25^{27-x}$ (Be specific for this problem.)

8. It is estimated that 3.5% of the general population will live past their 90\textsuperscript{th} birthday. Consider a graduating class of 753 high school seniors.

a) What is the expected number of seniors who will live past their 90\textsuperscript{th} birthday? 26.4 or about 26

b) What is the probability that 30 or more seniors will live past their 90\textsuperscript{th} birthday? 0.260

c) What is the probability that 25 to 35 seniors will live past their 90\textsuperscript{th} birthday? 0.594

d) What number of seniors who live past their 90\textsuperscript{th} birthday would be unusually high? 37 or more

e) What number of seniors who live past their 90\textsuperscript{th} birthday would be unusually low? 16 or less

9. It is estimated that 64% of people who are murdered actually knew the person who did the murder. Suppose that a detective file in Winnebago County has 62 unsolved murders.

a) What is the expected number of victims in the file who knew their murderer? about 40

b) What is the probability that at least 35 of the victims knew their murderer? 0.913

c) What is the probability that at most 48 of the victims knew their murderer? 0.992

d) What is the probability that fewer than 30 of the victims did not know their murderer? 0.970

e) What is the probability that more than 20 of the victims did not know their murderer? 0.681

$1 - P(r \leq 20) = \text{binomcdf}(62, .36, 20) = 0.681$