

Functions and Relations

Background: The concepts of function and relation are core concepts in mathematics. Ordinary folks use functions and relations every day without knowing they are using them!

Functions and Relations: All functions and relations have three components.

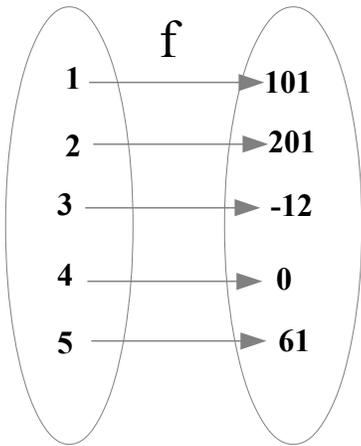
- 1) **Domain:** The domain of a function or relation can be any collection of objects. Elements in the domain can be numbers, matrices, people, cars, dogs, food items on a menu or anything one can imagine. The domain variable of a function or relation represents all possible input values and is said to be an **independent** variable.
- 2) **Range:** The range of a function or relation can be any collection of objects. Elements in the range can be numbers, matrices, people, cars, dogs, food items on a menu or anything one can imagine. The range variable of a function or relation represents all possible output values. Range variables are called **dependent** variables because their values depend on domain input values.
- 3) **Matching Rule:** The matching rule component tells us how to match domain objects with range objects. Matching rules are usually in the form of an equation, a table, list of ordered pairs, catalog, or set of directions. Every function is a relation, but not every relation is a function. If a relation is a function, every domain object is matched with only one range object. If a function is a one-to-one function, then any two different input values always have different output values. If **a** and **b** are domain input values, the symbols **f(a)** and **f(b)** represent the corresponding output values. If $f(x)$ is a one-to-one function and $f(a) = f(b)$, then $a = b$.

Examples of Functions and Relations:

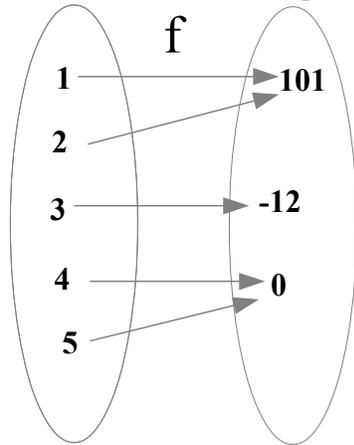
- a) **Restaurant Menu Function:** The domain of a restaurant menu equals all the food items for sale and the range equals the prices of the food items. Every food item of a specific type and size is matched with exactly one price. Since different food items may have the same price, a restaurant menu is not a one-to-one function. The matching rule is usually described on a sheet of heavy paper or on a chalkboard mounted on a wall.
- b) **Student Report Card Function:** The domain of a student's report card equals all courses the student was enrolled. The range of a report card equals all grades the student received for the courses enrolled. Students receive one grade for each course taken. Because students can receive the same grade in different courses, most student report cards are not one-to-one functions. The matching rule for a report card is usually described on a sheet of paper or in a data base that can be accessed at the school's web site..
- c) **People and Social Security Number Function:** The domain equals all people with a valid social security number and the range equals all valid social security numbers. Individuals can have only one valid social security number. The matching rule is determined and maintained by the Social Security Administration. This function is one-to-one because two different people who have valid social security numbers always have different social security numbers.
- d) **Cars and VIN Number Function:** The domain equals all legally manufactured cars and the range equals all of the vehicle identification numbers (VIN's) assigned to the cars. Every car has only one VIN and no two cars have the same VIN. The one-to-one matching rule is described in some computer data base.
- e) **People and Phone Number Relation:** The domain equals all people who have a phone and the range equals the phone numbers of all people who have a phone. Since an individual can have more than one phone number, the matching rule is a relation, but **not** a function.

f) **Function and Relation Diagrams:**

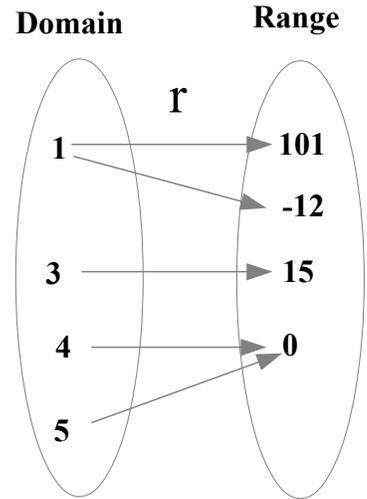
One-to-one function
Domain **Range**



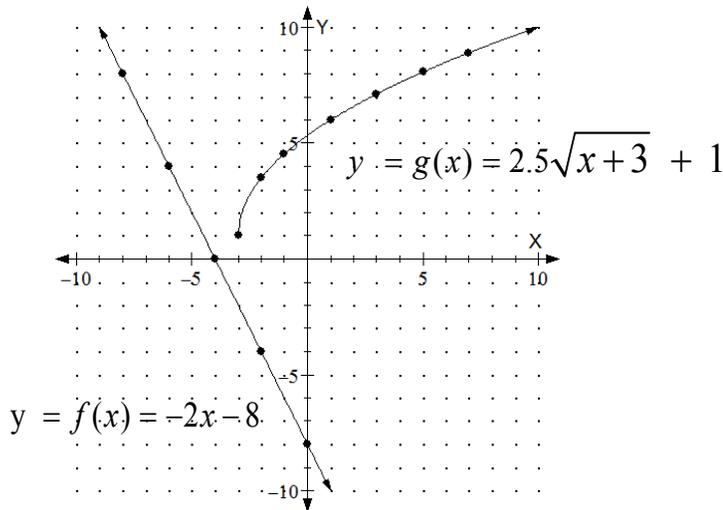
Function, but not one-to-one
Domain **Range**



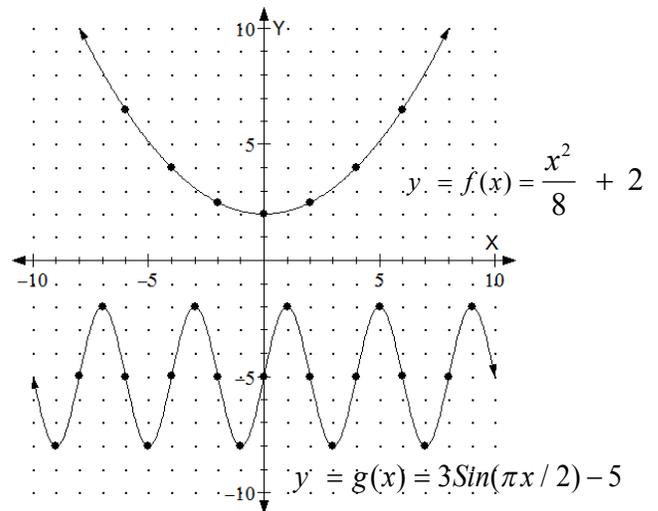
Relation, but not a function



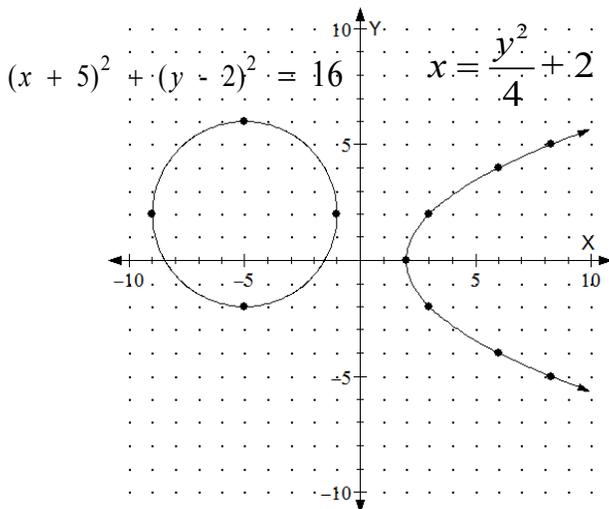
g) Two one-to-one functions of x .
 Both graphs pass the horizontal line test.



h) Two functions of x , but neither function is one to-one because both graphs fail the horizontal line test.



i) Two relations, but not functions of x .
 Both graphs fail the vertical line test.



j) Three variable function $z = f(x, y)$. Function inputs are x - y pairs of numbers and function outputs are single numbers. Function is not one-to-one because different input pairs have the same output value for z .

