

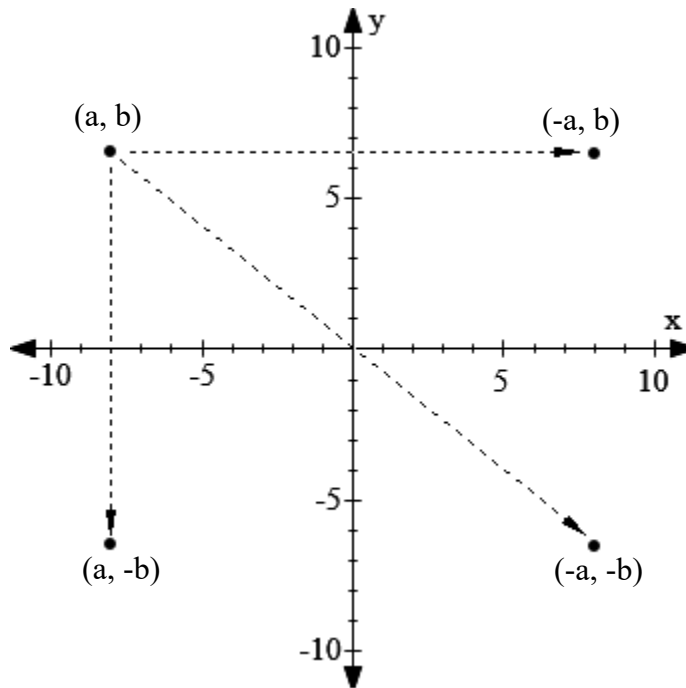
## Even and Odd Functions

In the the diagram below:

$(-a, b)$  is the reflection image of point  $(a, b)$  over the  $y$ -axis.

$(a, -b)$  is the reflection image of point  $(a, b)$  over the  $x$ -axis.

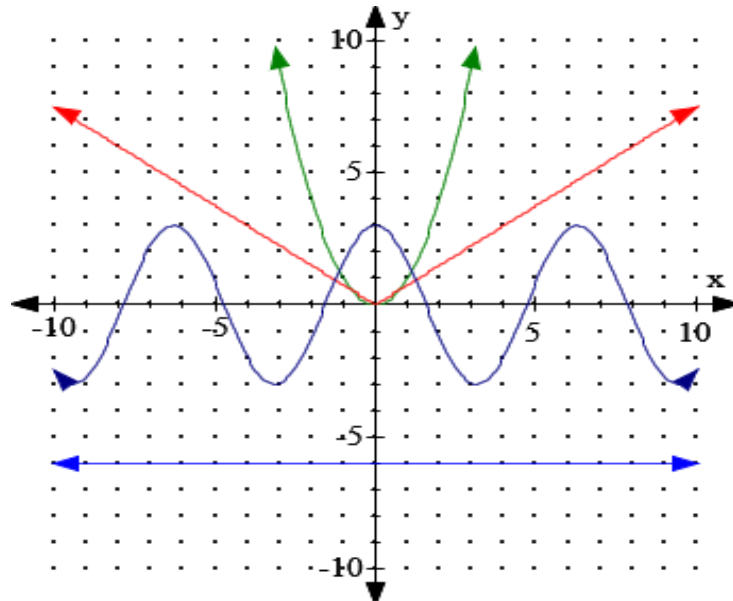
$(-a, -b)$  is obtained by rotating the point  $(a, b)$   $180^\circ$  about the origin.



Algebraic and Geometric Descriptions of Even Functions:

- A function is even if and only if  $f(-x) = f(x)$  for every  $x$  in the domain of the function.
- If point  $(a, b)$  is on the graph of an even function  $f(x)$ , then point  $(-a, b)$  is on the graph of  $f(x)$ .
- A function  $y = f(x)$  is even if and only if the equations  $y = f(x)$  and  $y = f(-x)$  are algebraically equivalent.
- A function  $y = f(x)$  is even if and only if its graph is symmetric with respect to the  $y$ -axis.

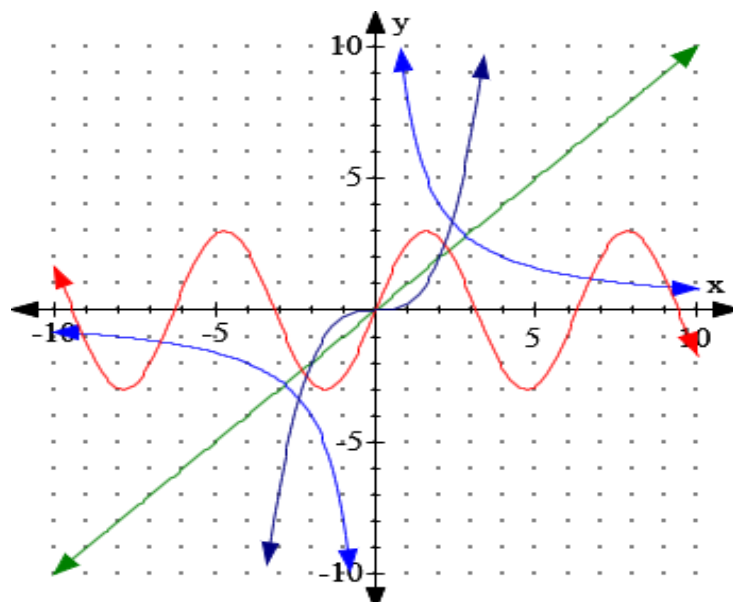
The functions  $y = x^2$ ,  $y = 3\cos(x)$ ,  $y = 0.75|x|$ , and  $y = -6$  below are all even functions.



#### Algebraic and Geometric Descriptions of Odd Functions:

- A function is odd if and only if  $f(-x) = -f(x)$  for every  $x$  in the domain of the function.
- If point  $(a, b)$  is on the graph of an odd function  $f(x)$ , then point  $(-a, -b)$  is on the graph of  $f(x)$ .
- A function  $y = f(x)$  is odd if and only if the equations  $y = f(x)$  and  $-y = f(-x)$  are algebraically equivalent.
- A function  $y = f(x)$  is odd if and only if its graph is symmetric with respect to the origin.

The functions  $y = x$ ,  $y = 3\sin(x)$ ,  $y = 8/x$ , and  $y = 0.25x^3$  below are all odd functions.



## Properties of Even and Odd Functions.

- \* The function  $y = k$  where  $k$  is a constant is an even function.  
Examples: The functions  $y = 6$  and  $y = -4.2$  are even functions.
- \* The function  $y = |x|$  is an even function.
- \* Functions of the form  $y = x^n$  where  $n$  is an even integer are even functions.  
Examples:  $y = x^2$ ,  $y = x^8$  and  $y = x^{-4}$  are even functions.
- \* Functions of the form  $y = x^n$  and  $y = x^{1/n}$  where  $n$  is an odd integer are odd functions.  
Examples:  $y = x$ ,  $y = x^3$ ,  $y = x^{1/3}$ ,  $y = x^{-5}$  and  $y = x^{-1/5}$  are odd functions.
- \* Functions of the form  $y = x^{1/n}$  where  $n$  is an even integer are neither even or odd.  
Examples:  $y = \sqrt{x} = x^{1/2}$ ,  $y = x^{1/4}$  and  $y = x^{-1/6}$  are neither even or odd.
- \* The function  $y = \text{Cos}(x)$  is an even function.
- \* The function  $y = \text{Sin}(x)$  is an odd function.
  
- \* The product of two even functions is even.  
Example:  $y = 13\text{Cos}(2x)\text{Cos}(3x)$  is even.
- \* The product of two odd functions is even.  
Example:  $y = x\text{Sin}(x)$  is even.
- \* The product of an even function and an odd function is odd.  
Example:  $y = x\text{Cos}(x)$  is odd.
  
- \* The quotient of two even functions is even.  
Example:  $y = 3/\text{Cos}(x)$  is even.
- \* The quotient of two odd functions is even.  
Example:  $y = x/\text{Sin}(x)$  is even.
- \* The quotient of an even function and an odd function is odd.  
Example:  $y = 4/x$  is odd.
  
- \* The sum or difference of two even functions is even.  
Example:  $y = x^2 + 0.3\text{Cos}(x)$  is even.
- \* The sum or difference of two odd functions is odd.  
Example:  $y = x - \text{Sin}(x)$  is odd..
- \* The sum or difference of an even function and an odd function is neither even or odd.  
Example:  $y = \text{Cos}(x) + \text{Sin}(x)$  is neither odd or even.
  
- \* Linear combinations of even functions are even.  
Example:  $y = -2x^4 + x^{-2} + 5\text{Cos}(3x) + 3$  is even.
- \* Linear combinations of odd functions are odd.  
Example:  $y = -2x^3 + x^{1/3} + 4\text{Sin}(3x)$  is odd.
  
- \* The composition two even functions is even.  
Example:  $y = \text{Cos}(x^2)$  is even.
- \* The composition of two odd functions is odd.  
Example:  $y = \text{Sin}(x^3)$  is odd.
- \* The composition of an even function and an odd function is even:  
Example: The function  $y = \text{Sin}(x^2)$  is even
- \* The composition of an odd function with an even function is even.  
Example: The function  $y = \text{Cos}(x^3)$  is even.