Basics of Solving Inequality Relations

Linear Inequalities (without absolute value function) :

Linear inequalities and linear equality equations are solved in a similar manner. When solving linear inequalities, one needs to understand when the sense of the inequality changes. <u>The sense of an inequality changes or switches **only** when both sides of the inequality are multiplied or divided by a negative number</u>.

Examples:

(Linear equality equation)

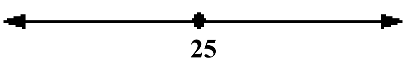
$3(x + 20) - 4x = x + 10$

$3x + 60 - 4x = x + 10$

$-x + 60 = x + 10$

$-2x = -50$

$x = 25$



(Linear inequality relation)

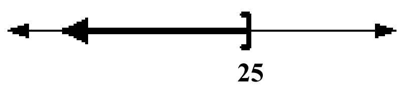
$3(x + 20) - 4x \geq x + 10$

$3x + 60 - 4x \geq x + 10$

$-x + 60 \geq x + 10$

$-2x \geq -50$ — Sense of inequality switches.

$x \leq 25$



(Linear inequality relation)

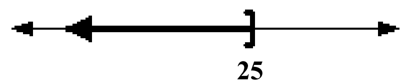
$3(x + 20) - 4x \geq x + 10$

$3x + 60 - 4x \geq x + 10$

$-x + 60 \geq x + 10$

$50 \geq 2x$

$25 \geq x$ — Sense of inequality does **not** switch.



(Compound linear inequality)

$-10 < 5x + 15 \leq 60$

$-25 < 5x \leq 45$

$-5 < x \leq 9$

($x > -5$ **and** $x \leq 9$)

(Compound linear inequality)

$-10 < -5x + 15 \leq 60$

$-25 < -5x \leq 45$

$5 > x \geq -9$ — Sense of inequality switches.

$-9 \leq x < 5$

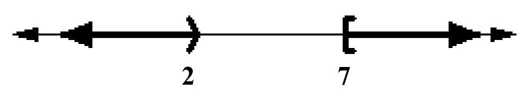
($x \geq -9$ **and** $x < 5$)

(Compound linear inequality)

$3x + 2 < 8$ **or** $2x - 3 \geq 11$

$3x < 6$ **or** $2x \geq 14$

$x < 2$ **or** $x \geq 7$



Linear Inequalities (with absolute value function) :

Linear inequalities that involve the absolute value function should first be rewritten as an equivalent compound inequality. If the absolute value of a quantity is less than a positive constant **k**, the quantity is between **-k** and **k**. If the absolute value of a quantity is more than a positive constant **k**, the quantity is less than **-k** **or** more than **k .** Recall that the absolute value of a quantity is the nonnegative distance of the quantity from zero.

Examples:

$|5x - 3| \leq 42$

$-42 \leq 5x - 3 \leq 42$

$-39 \leq 5x \leq 45$

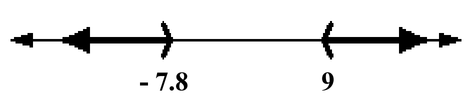
$-7.8 \leq x \leq 9$

$|5x - 3| > 42$

$5x - 3 < -42$ **or** $5x - 3 > 42$

$5x < -39$ **or** $5x > 45$

$x < -7.8$ **or** $x > 9$



Comment:

$|5x - 3| \leq -42$ has **no** solutions? Why?

$|5x - 3| \geq -42$, **any** real number is a solution! Why?

Nonlinear Inequalities Involving Polynomial and Rational Polynomial Expressions :

General Procedure:

a) Set one side of the inequality equal to zero by adding or subtracting the same quantity to/from both sides of the inequality. You may multiply or divide both sides of the inequality by a nonzero constant, but you **may not** multiply or divide both sides of an inequality by a quantity that contains a variable. Dividing both sides of an inequality by an expression that contains a variable will result in the destruction of solutions of the original inequality.

b) Write the expression as a polynomial or rational polynomial expression over a single denominator. Factor the polynomial or factor the numerator and denominator of the rational polynomial expression.

c) Find all **critical points** of the expression which are whose values of the variable for which the expression equals zero **or** the expression is undefined. The critical points determine the endpoints of test point intervals on the real number line.

d) For each test point interval, pick a test point between the endpoints of the interval. Use the test point to determine whether the expression is positive or negative in the test point interval.

e) Use the results from part (d) above to graph the solutions of the inequality and write an inequality that describes the graph of the inequality solutions.

Examples:

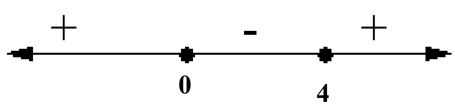
$x^2 \geq 4x$

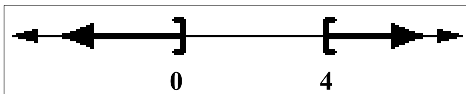
$x^2 - 4x \geq 0$

Subtract **4x** from both sides. Do **NOT** divide both sides by **x**.

$x(x - 4) \geq 0$

Critical points: x = 0 **or** 4 .

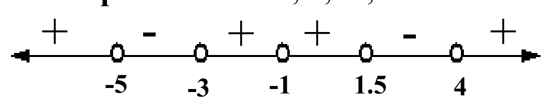


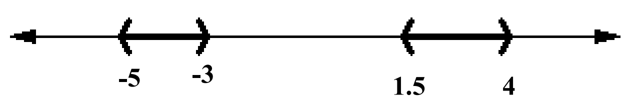


$x \leq 0$ **or** $x \geq 4$

$(2x - 3)(x - 4)(x + 1)^2(x + 3)(x + 5) < 0$

Critical points: x = 3/2, 4, -1, -3 or - 5

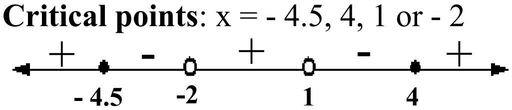


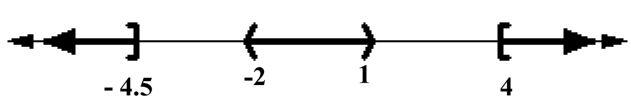


(-5 < x < 3) **or** (1.5 < x < 4)

$$\frac{(2x + 9)(x - 4)}{(x - 1)(x + 2)} \geq 0$$

Critical points: x = - 4.5, 4, 1 or - 2





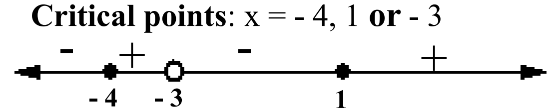
($x \leq$ - 4.5) **or** (-2 < x < 1) **or** ($x \geq 4$)

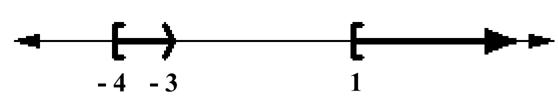
$$x \geq \frac{4}{x + 3}$$

$$x - \frac{4}{x + 3} \geq 0 \Rightarrow \frac{x^2 + 3x - 4}{x + 3} \geq 0$$

$$\frac{(x + 4)(x - 1)}{(x + 3)} \geq 0$$

Critical points: x = - 4, 1 **or** - 3





(-4 $\leq$ x < -3) **or** ($x \geq 1$)

Double Compound Inequalities : These type of inequalities can be solved like the standard absolute value inequalities discussed above, however, great care needs to be exercised in rewriting the inequality and interpreting results. (**and** operator means take the intersection of the sets. **or** operator means take the union of the sets.)

Example:

$2 < |2x-4| \le 10$

[$|2x-4| > 2$] **and** [$|2x-4| \le 10$]

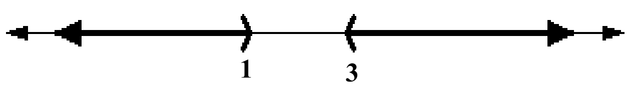
[$2x-4 < -2$ **or** $2x - 4 > 2$] **and** [$-10 \le 2x-4 \le 10$]

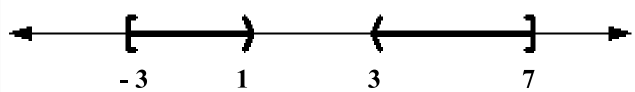
[$2x < 2$ **or** $2x > 6$] **and** [$-6 \le 2x \le 14$]

[$x < 1$ **or** $x > 3$] **and** [$-3 \le x \le 7$]

Comment: When parents give their child a choice, they use the word **or**. The child gets his choice of one thing or another thing, but not both. This meaning of the word **or** is called <u>exclusive or</u>.

When dealing with inequalities, the word or means one thing or the other thing or possibly both. This meaning of the word or is called <u>inclusive or.</u>





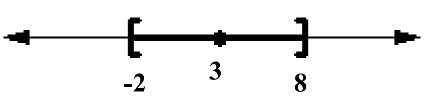
$(-3 \le x < 1)$ **or** $(3 < x \le 7)$

Inequalities of the form: | x - c | < k or |x - c | > k where **c** is any constant and **k** is a positive constant. These types of inequalities can be solved by inspection if you understand that **| x – c |** equals the positive distance from **c** to **x** . For < and ≤ inequalities, **c** is the center of the solution interval and **k** is the radius of the solution interval.

Examples:

x is any real number such that the distance from 3 to x is 5 or less.

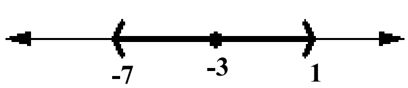
$|x-3| \le 5$



$-2 \le x \le 8$

x is any real numbers such that the distance from -3 to x is less than 4.

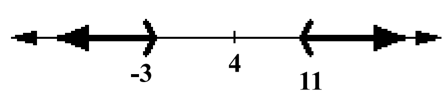
$|x+3| < 4$



$-7 < x < 1$

x is any real numbers such that the distance from 4 to x is more than 7.

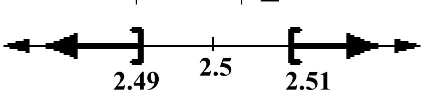
$|x-4| > 7$



$x < -3$ **or** $x > 11$

Let **x** = the diameter of a machine part. The part will fail inspection if its diameter deviates from 2.5 cm by .01 cm or more.

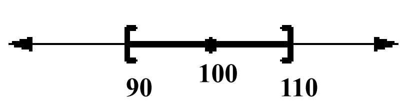
$|x - 2.5| \ge .01$



$x \le 2.49$ **or** $x \ge 2.51$

Let **x** = the IQ score of a normal person. The IQ score of a normal person deviates from 100 by no more than 10 points .

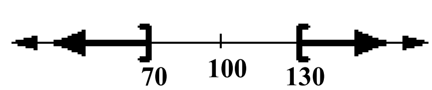
$|x - 100| \le 10$



$90 \le x \le 110$

Let x = the IQ score of a special person. The IQ score of a special person deviates from 100 by 30 or more points.

$|x - 100| \ge 30$



$x \le 70$ **or** $x \ge 130$