Basic Powers of Numbers and Miscellaneous Math Facts

The squares, cubes, powers of 2 and powers of 10 listed below should be committed to memory. Instant recall of these facts will make working with exponents and factoring of polynomials much easier.

Squares		Cubes		Powers of 2	Powers of 10
$1^2 = 1$	$9^2 = 81$	$1^3 = 1$	$9^3 = 729$	$2^1 = 2$	$10^3 = 1{,}000$ (1 k = 1 kilo)
$2^2 = 4$	$10^2 = 100$	$2^3 = 8$	$10^3 = 1{,}000$	$2^2 = 4$	$10^6 = 1{,}000{,}000$ (1 M = 1 mega)
$3^2 = 9$	$11^2 = 121$	$3^3 = 27$		$2^3 = 8$	$10^9 = 1{,}000{,}000{,}000$ (1 G = 1 giga)
$4^2 = 16$	$12^2 = 144$	$4^3 = 64$		$2^4 = 16$	$10^{-1} = 0.1$ (1 d = 1 deci)
$5^2 = 25$	$13^2 = 169$	$5^3 = 125$		$2^5 = 32$	$10^{-2} = 0.01$ (1 c = 1 centi)
$6^2 = 36$	$14^2 = 196$	$6^3 = 216$		$2^6 = 64$	$10^{-3} = 0.001$ (1 m = 1 milli)
$7^2 = 49$	$15^2 = 225$	$7^3 = 343$		$2^{10} = 1{,}024$ (computer - 1 K = 1,024)	$10^{-6} = 0.000\ 001$ (1 μ = 1 micro)
$8^2 = 64$	$16^2 = 256$	$8^3 = 512$		$2^{20} = 1{,}048{,}576$ (computer - 1 meg = 1,048,576)	$10^{-9} = 0.000\ 000\ 001$ (1 n = 1 nano)

Addition Table for Basic Addition Facts

+	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13
2	3	4	5	6	7	8	9	10	11	12	13	14
3	4	5	6	7	8	9	10	11	12	13	14	15
4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	7	8	9	10	11	12	13	14	15	16	17
6	7	8	9	10	11	12	13	14	15	16	17	18
7	8	9	10	11	12	13	14	15	16	17	18	19
8	9	10	11	12	13	14	15	16	17	18	19	20
9	10	11	12	13	14	15	16	17	18	19	20	21
10	11	12	13	14	15	16	17	18	19	20	21	22
11	12	13	14	15	16	17	18	19	20	21	22	23
12	13	14	15	16	17	18	19	20	21	22	23	24

Multiplication Table for Basic Multiplication Facts

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Important Factoring Patterns and Techniques for Factoring Polynomials

1. **First look for a common factor** and factor out the greatest common factor. If the leading coefficient is negative, factor out a negative one (-1). You may have to first rearrange the polynomial so that the terms of the polynomial are in descending order by the degree of the terms.

   Examples: $2x^3 - 14x^2 + 10x = 2x(x^2 - 7x + 5)$; $-15xy^4 - 20xy^3 + 5xy^2 = -5xy^2(3y^2 + 4y - 1)$

   $x(\mathbf{2x + y}) - 3(\mathbf{2x + y}) = (\mathbf{2x + y})(x - 3)$; $y^3(\mathbf{y + 1}) + (\mathbf{y + 1}) = (\mathbf{y + 1})(y^3 + 1)$

2. **If** the polynomial is a binomial, check to see if it is the

   a) difference of two squares: $a^2 - b^2 = (a + b)(a - b)$ Ex. $16t^2 - 25 = (4t + 5)(4t - 5)$
   Ex. $z^4 - 36w^2 = (z^2 + 6w)(z^2 - 6w)$

   b) sum of two squares: $a^2 + b^2$ can not be factored. Ex. $16t^2 + 25$ is prime, it can't be factored.

   c) sum of two cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ Ex. $27a^3 + 64b^3 = (3a + 4b)(9a^2 - 12ab + 16b^2)$
   Ex. $8t^3 + 1 = (2t + 1)(4t^2 - 2t + 1)$

   d) difference of two cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ Ex. $125x^3 - 1 = (5x - 1)(25x^2 + 5x + 1)$
   Ex. $1{,}000y^3 - 343z^3 = (10y - 7z)(100y^2 + 70yz + 49z^2)$

3. **If** the polynomial is a trinomial, check to see if it is a

   a) perfect trinomial square: $a^2 + 2ab + b^2 = (a + b)(a + b)$ Ex. $4x^2 + 12xy + 9y^2 = (2x + 3y)(2x + 3y)$
   Ex. $25a^2 + 40ab + 16y^2 = (5a + 4b)(5a + 4b)$

   $a^2 - 2ab + b^2 = (a - b)(a - b)$ Ex. $25p^2 - 40pq + 16q^2 = (5p - 4q)(5p - 4q)$
   Ex. $9c^2 - 60cd + 100d^2 = (3c - 10d)(3c - 10d)$

   b) general trinomial: Use reverse FOIL to factor the polynomial. If you don't recognize the perfect trinomial pattern, don't worry, just try reverse FOIL to factor the trinomial.
   Ex. $2t^2 - 5t + 2 = (2t - 1)(t - 2)$
   Ex. $6y^3 + 19y^2 - 7y = \mathbf{y}(6y^2 + 19y - 7) = \mathbf{y}(3y - 1)(2y + 7)$

4. **If** the polynomial has four or more terms, try to **factor by grouping**. Factor each subgroup and then check to see if any of the factored subgroups have any common factors. If the polynomial has four terms, consider grouping the terms as two binomials **or** as a trinomial and a monomial.

   Ex. $\boxed{x^2 - 2x} + \boxed{xy - 2y} = x(\mathbf{x - 2}) + y(\mathbf{x - 2}) = (\mathbf{x - 2})(x + y)$

   Ex. $\boxed{9x^2 - 6x + 1} - \boxed{y^2} = (3x - 1)(3x - 1) - \mathbf{y}^2 = (3x - 1)^2 - \mathbf{y}^2 = (3x - 1 + \mathbf{y})(3x - 1 - \mathbf{y})$
   $= (3x + y - 1)(3x - y - 1)$

5. **Always** check to see if all factors are prime. Don't stop factoring until all factors are prime.
   Example: $81a^4 - 16 = (9a^2 + 4)(9a^2 - 4) = (9a^2 + 4)(3a + 2)(3a - 2)$

Common Decimal Equivalents

$\frac{1}{8} = 0.125$	$\frac{1}{10} = 0.1$	$\frac{1}{6} = 0.166666...$	$\frac{1}{9} = 0.111111...$
$\frac{1}{4} = 0.250$	$\frac{1}{5} = 0.2$	$\frac{1}{3} = 0.333333...$	$\frac{2}{9} = 0.222222...$
$\frac{3}{8} = 0.375$	$\frac{3}{10} = 0.3$	$\frac{1}{2} = 0.500000$	$\frac{1}{3} = 0.333333...$
$\frac{1}{2} = 0.500$	$\frac{2}{5} = 0.4$	$\frac{2}{3} = 0.666666...$	$\frac{4}{9} = 0.444444...$
$\frac{5}{8} = 0.625$	$\frac{5}{10} = 0.5$	$\frac{5}{6} = 0.833333...$	$\frac{5}{9} = 0.555555...$
$\frac{3}{4} = 0.750$	$\frac{3}{5} = 0.6$	$\frac{6}{6} = 1.000000$	$\frac{2}{3} = 0.666666...$
$\frac{7}{8} = 0.875$	$\frac{7}{10} = 0.7$		$\frac{7}{9} = 0.777777...$
$\frac{8}{8} = 1.000$	$\frac{4}{5} = 0.8$		$\frac{8}{9} = 0.888888...$
	$\frac{9}{10} = 0.9$		$\frac{9}{9} = 1.000000$
	$\frac{10}{10} = 1.0$		

Slope, Distance and Midpoint Formulas

Let **A**(x_1, y_1) and **B**(x_2, y_2) be two points in the x-y coordinate plane.

Slope of line that contains points **A** and **B** $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_1 \neq x_2)$

Distance between points **A** and **B** $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

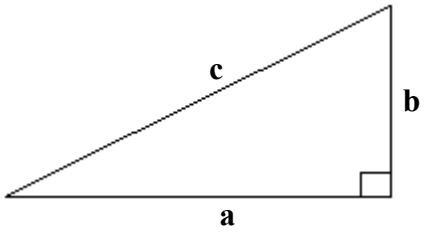
Midpoint of the line segment with endpoints at **A** and **B** $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Pythagorean Theorem- The sum of the squares of the two shortest sides of a triangle equals the square of the longest side if and only if the triangle is a **right** triangle.

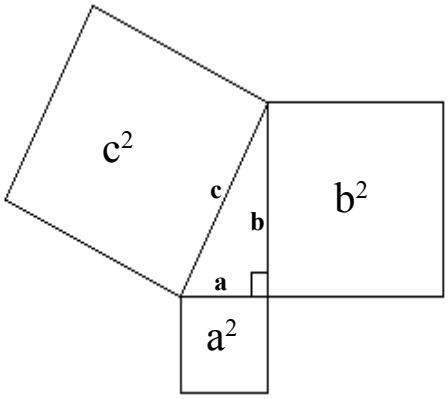
$$\mathbf{a^2 + b^2 = c^2}$$



All **Pythagorean triples** in which the hypotenuse is less than 100 and the lengths of the sides are relatively prime. Students should memorize the boxed triples.

3 – 4 – 5	16 – 63 - 65
5 – 12 – 13	20 – 21 – 29
8 – 15 – 17	28 – 45 – 53
7 – 24 – 25	33 – 56 – 65
9 – 40 – 41	36 – 77 – 85
11 – 60 – 61	39 – 80 - 89
12 – 35 – 37	48 – 55 – 73
13 – 84 - 85	65 – 72 – 97

Comment: The Babylonians knew of hundreds of triples for the sides of a right triangle by 2000 BC They found that the triple (3,367, 3,456, 4,825) formed the sides of a right triangle. It was not until around 550 BC that Pythagoras first proved the relationship for the sides of a right triangle. There are now more than 300 proofs of this famous and most important theorem in all of mathematics.



Order of Operations

The value of a mathematical expression depends on the order in which the operations are performed. The sentence "Please Excuse My Dear Aunt Sally" is a helpful device to remember the order of operation rules.

P represents **all** grouping symbols such as ( ) , [ ], { }, $\sqrt{\text{exp}ression}$, | |, and $\overline{\qquad}$ (division bar)

Always do the operations inside the grouping symbols **first**.

E represents all exponential operations such as squaring a number, cubing a number and raising a number to any power. After all of the grouping symbols are removed, do the exponential operations next.

M and **D** represent the operations of multiplication and division. After all grouping symbols are removed and all exponential operations performed, do all multiplication and division operations next. Always work left to right. Do what ever operation comes first.

A and **S** represent the operations of addition and subtraction. After all grouping symbols are removed, all exponential operations performed, and all multiplication and division operations performed, do the addition and subtraction operations next**.** Always work left to right. Do what ever operation comes first**.**

Properties of Real Numbers

Commutative properties: a + b = b + a a*b = b*a

(The order in which two numbers are added or multiplied makes **no** difference.)

Associative properties: a + (b + c) = (a + b) + c a*(b * c) = (a*b)*c

(How three numbers are grouped for addition or multiplication makes **no** difference.)

Distributive properties: a(b + c) = ab + ac a(b – c) = ab – ac

(Multiplication distributes over addition.) (Multiplication distributes over subtraction.)

$\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$ $a \neq 0$

(Division distributes over addition.)

$\frac{b-c}{a} = \frac{b}{a} - \frac{c}{a}$ $a \neq 0$

(Division distributes over subtraction.)

Identity Properties: a + **0** = **0** + a = a **1***a = a***1** = a

(The additive identity element is **0**.) (The multiplicative identity element is **1**.)

Inverse Properties: a + (-a) = -a + a = **0**

(The **sum** of a number and the **opposite** of the number always equals the additive identity element **0**.)

$a * \frac{1}{a} = \frac{1}{a} * a = 1$ $a \neq 0$

(The **product** of a number and the reciprocal of the number always equals the multiplicative identity element **1**.)

Comment: The **additive inverse** of a number equals the **opposite** of a number and the **multiplicative inverse** of a number equals the **reciprocal** of the number.