Basic Equation Solving Strategies

Case 1: The variable appears only <u>once</u> in the equation. (Use work backwards method.)

1. Simplify both sides of the equation if possible.
2. Apply the order of operations rules to make an **equation solve plan**.
3. Starting with the <u>last step</u> in the solve plan, apply the same operation to <u>both sides</u> of the equation that reverses the step in the solve plan. Continue <u>working backwards</u> <u>towards the variable being solved</u> for by reversing or undoing each step in the solve plan.
4. Check the solution. Substitute each solution into the <u>original equation</u> to see if the resulting equation is true.

Basic Operation and the Corresponding Reverse Operation

a) + <---------> - (Addition and subtraction reverse each other.)

b) * <----------> / (Multiplication and division reverse each other.)

c) x^2 <----------> √ **or** -√ (Raising to a power and root operation reverse each other.)

x^3 <-----------> $\sqrt[3]{\ }$

Example 1: If $w^2 = 121$, then $w = 11$ or $w = -11$.

Example 2: If $(y-3)^2 = 289$, then $y - 3 = 17$ **or** $y - 3 = -17$

$y = 20$ **or** $y = -14$.

Example 3: If $(y - 3)^2 = -289$, then $(y-3)^2$ can **NOT** be reversed! Why?

Example 4: If $(x-2)^3 = 125$, then $x - 2 = 5$ ====> $x = 7$

Example 5: If $(x-2)^3 = -125$, then $x - 2 = -5$ ====> $x = -3$

d) |x| <----------> x **or** -x (x and -x have the same absolute value.)

Example 1: If $|t| = 12$, then $t = 12$ **or** $t = -12$.

Example 2: If $|4w| = 20$, then $4w = 20$ **or** $4w = -20$

Example 3: If $|4w| = -20$, then $|4w|$ can **NOT** be reversed!! Why?

e) -x <---------> -x (Taking the opposite of a number is its own reverse operation.)

1/x <--------> 1/x (Taking the reciprocal of a number is its own reverse operation.)

Ex. 1 $30x + 50 = 770$

Solve plan: x, *30, + 50

$30x = 720$ (- 50)

$\mathbf{x = 24}$ (/ 30)

Ex. 2 $\frac{4x - 17}{10} = -20$

Solve plan: x, * 4, -17, / 10

$4x - 17 = -200$ (* 10)

$4x = -183$ (+ 17)

$\mathbf{x = -45.75}$ (/ 4)

Ex. 3 $\frac{4y}{10} - 17 = -20$

Solve plan: x, * 4, / 10, - 17

$\frac{4y}{10} = -3$ (+ 17)

$4y = -30$ (* 10)

$\mathbf{y = -7.5}$ (/ 4)

Ex. 4 $\frac{\sqrt{5t+9}}{3} - 10 = -4$

t
* 5
+ 9
√x
/ 3
-10

$\frac{\sqrt{5t+9}}{3} = 6$ (+10)

$\sqrt{5t+9} = 18$ (*3)

$5t + 9 = 324$ (x^2)

$5t = 315$ (-9)

t = 63 (/5)

Ex. 5 $(2y - 7)^2 + 1 = 290$

y
* 2
- 7
x^2
+ 1

$(2y - 7)^2 = 289$ (-1)

$2y - 7 = 17$ **or** $2y - 7 = -17$ ($\pm\sqrt{\ }$)

$2y = 24$ or $2y = -10$ (+ 7)

y = 12 or y = -5 (/ 2)

Ex. 6 $\frac{|4w - 3|}{7} = 60$

w
* 4
- 3
| |
/ 7

$|4w - 3| = 420$ (* 7)

$4w - 3 = 420$ **or** -420 (Undo | |)

$4w = 423$ **or** -417 (+ 3)

w = 105.75 or -104.25

Case 2: The variable appears more than once in the equation and is raised to the same power through out the equation.

1. **Simplify both sides** of the equation if possible. Use the distributive property to remove parentheses and combine like terms. At this stage you are only simplifying each side of the equation separately.
2. **Gather the variable** on the same side of the equation by adding or subtracting a multiple of the variable to both sides of the equation. (**Note:** x, 2x, 3x, 4x, 5x, 6x, . . . are multiples of x .)
3. The variable should now appear only once in the equation. Use the work backwards method to solve the equation.
4. Check your solution. Substitute each solution into the original equation to see if the resulting equation is true.

Ex. 1 $5(x - 4) - x + 12 = 2(x + 10) - 8$

$5x - 20 - x + 12 = 2x + 20 - 8$ (Distributive prop.)

$4x - 8 = 2x + 12$ (Simplify both sides of equation.)

x
*2
- 8

$2x - 8 = 12$ (Gather variable, subtract 2x)

$2x = 20$ (Add 8)

x = 10 (Divide by 2)

Ex. 2 $\frac{6x + 18}{3} = -3(x - 8) - 4$

$2x + 6 = -3x + 24 - 4$ (Dist. prop. to simplify)

$2x + 6 = -3x + 20$ (Simplify right side.)

x
* 5
+ 6

$5x + 6 = 20$ (Gather variable, add 3x)

$5x = 14$ (Subtract 6)

x = 2.8 (Divide by 5)

Ex. 3 $5x - (10 - 2x) + 12 = 3x - (x - 20)$

$5x - 10 + 2x + 12 = 3x - x + 20$ (Remove ( ))

$7x + 2 = 2x + 20$ (Simplify both sides)

x
* 5
+ 2

$5x + 2 = 20$ (Gather variable, subtract 2x)

$5x = 18$

x = 18/5 = **3.6**

Ex. 4 $y^2 + y^2 + y^2 + y^2 - 10 = 186$

y
x^2
* 4
-10

$4y^2 - 10 = 186$ (Simplify left side of the equation.)

$4y^2 = 196$

$y^2 = 49$

y = **7** **or** y = **-7**

Case 3: The equation is a simple proportion. (Use the cross product property of fractions.)

1. Simplify both sides of the equation if possible.
2. Use the **cross product property** of fractions or proportions to rewrite the equation in equivalent form.
3. Solve the resulting equation by using the methods discussed above.
4. Check your solution. Substitute each solution into the original equation to see if the resulting equation is true.

Ex 1: $\frac{y-3}{y+6} = \frac{4}{7}$

$7y - 21 = 4y + 24$

$3y - 21 = 24$

$3y = 45$

$\mathbf{y = 15}$ ✓

Ex. 2: $\frac{3}{x-5} = \frac{4}{x}$

$3x = 4x - 20$

$-x = -20$

$\mathbf{x = 20}$ ✓

Ex. 3: $\frac{w}{18} = \frac{2}{w}$

$w^2 = 36$

$\mathbf{w = 6}$ **or** $\mathbf{-6}$ ✓ ✓

Ex. 4: $\frac{2x}{x-2} = 4$ (Think **4** over 1)

$2x = 4x - 8$

$-2x = -8$

$\mathbf{x = 4}$ ✓

Case 4: The equation involves different powers of the variable. (Try the factoring method.)

1. Set one side of the equation equal to **zero** by adding or subtracting algebraic expressions to both sides of the equation.
2. If necessary, simplify and rewrite the equation so that it is easier to factor.
3. **Factor** the equation.
4. Set each factor that contains a variable to zero and solve each of the resulting equations.
5. Each solution obtained in step (4) above should be a solution to the original equation.
6. Check all solutions with the original equation. This will help you to better understand what you have found and develop the basic skill of evaluating expressions.

Ex. 1 $16t^3 = 64t^2$

$16t^3 - 64t^2 = 0$

(Factor out common factor of $\mathbf{16t^2}$).

$16t^2(t - 4) = 0$

$16t^2 = 0$ **or** $t - 4 = 0$

$t^2 = 0$ **or** $t = 4$

$\mathbf{t = 0}$ **or** $\mathbf{t = 4}$ ✓ ✓

Ex. 2 $3x^2 - 6x = 45$

$3x^2 - 6x - 45 = 0$

$3(x^2 - 2x - 15) = 0$

(Factor out common factor of **3**.)

$3(x - 5)(x + 3) = 0$

(Use reverse FOIL to factor trinomial.).

$x - 5 = 0$ **or** $x + 3 = 0$

$\mathbf{x = 5}$ **or** $\mathbf{x = -3}$ ✓ ✓

Ex. 3 $2x^3 = 8x^2 + 42x$

$2x^3 - 8x^2 - 42x = 0$

$2x(x^2 - 4x - 21) = 0$ (Factor out common factor of **2x**).

$2x(x - 7)(x + 3) = 0$ (Use reverse FOIL to factor trinomial.).

$2x = 0$ **or** $x - 7 = 0$ **or** $x + 3 = 0$

$\mathbf{x = 0}$ **or** $\mathbf{x = 7}$ **or** $\mathbf{x = -3}$ ✓ ✓ ✓

Ex. 4 $\frac{y}{4} = \frac{y-4}{y-6}$

$y^2 - 6y = 4y - 16$

$y^2 - 10y + 16 = 0$

$(y - 8)(y - 2) = 0$

$\mathbf{y = 8}$ **or** $\mathbf{y = 2}$ ✓ ✓

Ex. 5 $\frac{x^2(6x + 37)}{35} = x$ (Think **x** over 1)

$6x^3 + 37x^2 = 35x$

$6x^3 + 37x^2 - 35x = 0$

$x(6x^2 + 37x - 35) = 0$

(Factor out common factor of **x**.)

$x(6x - 5)(x + 7) = 0$

(Use reverse FOIL to factor trinomial.).

$x = 0$ **or** $6x - 5 = 0$ **or** $x + 7 = 0$

$\mathbf{x = 0}$ **or** $\mathbf{x = 5/6}$ **or** $\mathbf{x = -7}$ ✓ ✓ ✓

Ex. 6. $z^4 - 13z^2 + 36 = 0$

(Largest exponent is twice next largest.)

$(z^2 - 4)(z^2 - 9) = 0$

(Use reverse FOIL to factor trinomial.).

$(z + 2)(z - 2)(z + 3)(z - 3) = 0$

(Factor as difference of two squares.).

$\mathbf{z = -2}$ **or** $\mathbf{z = 2}$ **or** $\mathbf{z = -3}$ **or** $\mathbf{z = 3}$ ✓ ✓ ✓ ✓

Case 5: The equation involves fractions, but the equation is not a proportion.

1. Simplify each side of the equation if possible.
2. Find the lowest common denominator of all of the fractions that appear in the equation.
3. Multiple both sides of the equation by the lowest common denominator. This will produce an equation with no fractions!
4. Simplify each side of the equation by using the distributive property and then combine like terms.
5. Solve the resulting equation by using the methods discussed above.
6. **Check all solutions** with the original equation. Step (3) may have created solutions that do **not** satisfy the original equation.

Ex. 1: $\frac{1}{3}+\frac{2}{5}=\frac{1}{x}$

$$15x\left(\frac{1}{3}+\frac{2}{5}\right)=15x\frac{1}{x}$$

$$5x+6x=15$$

$$11x=15$$

$$x=15/11=1\frac{4}{11}$$ ✓

Ex. 2: $\frac{4}{3}+\frac{5}{x+2}=10$

$$3(x+2)\left(\frac{4}{3}+\frac{5}{(x+2)}\right)=3(x+2)10$$

$$4(x+2)+15=30(x+2)$$

$$4x+8+15=30x+60$$

$$4x+23=30x+60$$

$$23=26x+60$$

$$-37=26x$$

$$x=\frac{-37}{26}=-1.4230769$$ ✓

Ex. 3 $\frac{x+3}{x-1}=\frac{4}{x-1}$

$$(x-1)\left(\frac{x+3}{x-1}\right)=(x-1)\frac{4}{(x-1)}$$

$$x+3=4$$

$$x=1$$

Substituting 1 for x in the original equation results in division by zero!

$$\frac{4}{0}=\frac{4}{0}$$

Since the only possible solution is not a solution, there are NO solutions!!! ✓

Ex. 4 $\frac{4}{5}+y=\frac{4y-50}{5y-25}$

$\frac{4}{5}+y=\frac{4y-50}{5(y-5)}$ (Factor the denominator on the right side of equation. It is easier to find the lowest common denominator when all denominators are factored.)

$5(y-5)\left(\frac{4}{5}+y\right)=5(y-5)\frac{(4y-50)}{5(y-5)}$ (Multiply both sides of the equation by **5(y-5)** which is the lowest common denominator of all of the fractions in the equation.)

$4(y-5)+5y(y-5)=4y-50$ (Simplify both sides of the equation. We have **no more factions** to deal with!!)

$4y-20+5y^2-25y=4y-50$ (Use the distributive property to remove grouping symbols.)

$5y^2-21y-20=4y-50$ (Simplify the left side of the equation. We are dealing with a case 4 situation.)

$5y^2-25y+30=0$ (Set one side of the equation to zero.)

$5(y^2-5y+6)=0$ (Factor out the common factor of **5**.)

$5(y-2)(y-3)=0$ (Use reverse FOIL to factor the trinomial y^2-5y+6)

$y-2=0$ **or** $y-3=0$ (Set each factor that contains a variable equal to zero.)

y = 2 or y = 3 (Solve resulting equations. Both solutions check with the original equation.)

✓ ✓

Case 6: The equation is a 2nd degree polynomial equation. (Use the quadratic formula to solve the equation.)

Quadratic formula: If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex. 1 $x^2 - 2x = 15$

$x^2 - 2x - 15 = 0$ (Set the right side of the equation to **0**.)

$a = 1$, $b = -2$, and $c = -15$ (Determine the values of a, b and c.)

$x = \dfrac{2 \pm \sqrt{4 - 4(-15)}}{2}$ (Plug the values of a, b, and c into the quadratic formula.)

$x = \dfrac{2 + \sqrt{64}}{2}$ or $\dfrac{2 - \sqrt{64}}{2}$ (Use the order of operation rules to evaluate each part of the formula.)

$x = 5$ or $x = -3$ ✔✔

Comment: It would have been much easier to use the factoring method to solve this equation. If you can quickly factor the polynomial, then use the factoring method. Otherwise just bite the bullet and use the formula. The quadratic formula always works!

Ex. 2 $-16t^2 + 84t + 20 = 100$

$-16t^2 + 84t - 80 = 0$ (Set the right side of the equation to **0**.)

$a = -16$, $b = 84$, and $c = -80$ (Determine the values of a, b and c.)

$x = \dfrac{-84 \pm \sqrt{7{,}056 - 4(1{,}280)}}{-32}$ (Plug the values of a, b, and c into the quadratic formula.)

$x = \dfrac{-84 + \sqrt{1936}}{-32}$ or $x = \dfrac{-84 - \sqrt{1936}}{-32}$

$x = \dfrac{84 - \sqrt{1936}}{32}$ or $x = \dfrac{84 + \sqrt{1936}}{32}$ (Multiply by $\dfrac{-1}{-1}$)

(Multiply the numerator and denominator of each fraction by **-1** which will reduce the number of negative signs.)

$x = 1.25$ or $x = 4$ ✔ ✔

Ex. 3. Solve $y^4 + 12y^2 = 9$

At first glance it would appear that the quadratic formula could not be used on this equation. However, if the value of the **largest exponent** is is **twice** the value of the **lower variable exponent,** the equation can be rewritten so that the quadratic formula applies.

$y^4 + 12y^2 = 9$

$(y^2)^2 + 12(y^2)^1 - 9 = 0$

$a = 1$, $b = 12$, and $c = -9$ (Determine the values of a, b and c.)

$\mathbf{y^2} = \dfrac{-12 \pm \sqrt{144 - 4(-9)}}{2}$

$\mathbf{y^2} = \dfrac{-12 + \sqrt{180}}{2}$ or $\dfrac{-12 - \sqrt{180}}{2}$

$y^2 = 0.7082039$ or $y^2 = -12.7082$

$y^2 = 0.7082039$ ====> **$y = 0.8415$ or $y = -0.8415$** ✔ ✔

$y^2 = -12.7082$ =====> **$y = 3.5649\,i$ or $y = -3.5649\,i$** . ✔ ✔

Comment: If the situation dictates that real number solutions are the only solutions of interest, then ignore the two complex number solutions.

$i = \sqrt{-1}$ and $i^2 = -1$

Ex. 4 Solve $x^{2/3} - 7x^{1/3} + 12 = 0$

The strategy used in example **3** will be used to solve this equation.

$x^{2/3} - 7x^{1/3} + 12 = 0$

$(x^{1/3})^2 - 7(x^{1/3})^1 + 12 = 0$

$a = 1$, $b = -7$, and $c = 12$ (Determine the values of a, b and c.)

$x^{1/3} = \dfrac{7 \pm \sqrt{49 - 4(12)}}{2}$

$x^{1/3} = \dfrac{7 + \sqrt{1}}{2}$ or $\dfrac{7 - \sqrt{1}}{2}$

$x^{1/3} = \dfrac{7 + 1}{2}$ or $\dfrac{7 - 1}{2}$

$x^{1/3} = 4$ or 3

$x^{1/3} = 4$ ====> **$x = 64$** ✔ (1/3 power = cube root. So cube both sides.)

$x^{1/3} = 3$ ====> **$x = 27$** ✔

Ex. 5 Solve $4x^2 - 6x + 3 = 0$

$a = 4$, $b = -6$, and $c = 3$

$$x = \frac{6 \pm \sqrt{36 - 4(12)}}{8}$$

$$x = \frac{6 \pm \sqrt{-12}}{8}$$

$i = \sqrt{-1}$ and $i^2 = -1$

$(2\sqrt{3}i)(2\sqrt{3}i) = 12i^2 = -12$

$(12 = 4 * 3)$

$$x = \frac{6 + \sqrt{-12}}{8} \quad \text{or} \quad \frac{6 - \sqrt{-12}}{8}$$

$$x = \frac{6 + 2\sqrt{3}\,\mathbf{i}}{8} \quad \textbf{or} \quad \frac{6 - 2\sqrt{3}\,\mathbf{i}}{8}$$

$$x = \frac{3}{4} + \frac{\sqrt{3}}{4}\mathbf{i} \quad \textbf{or} \quad \frac{3}{4} - \frac{\sqrt{3}}{4}\mathbf{i}$$

The Discriminant

The quantity $\mathbf{b^2 - 4ac}$ used in the quadratic formula is called the **discriminant**. The discriminant reveals the nature of the roots and some properties of the parabolic graph.

1. If $\mathbf{b^2 - 4ac > 0}$ (positive) , the polynomial equation will have two different real roots and the graph of the parabola will cross the x-axis at two different points.

2. If $\mathbf{b^2 - 4ac = 0}$, the polynomial equation will have exactly one real number root and the vertex of the parabola will just touch the x-axis at one point.

3. If $\mathbf{b^2 - 4ac < 0}$ (negative) , the polynomial equation will have **no** real roots , but two complex number roots. The parabola will **not** cross the x-axis.

4. In examples 1 – 4 of this section the discriminant was positive and consequently each of these equations had two real roots. In example **5** the discriminant was negative and the equation had two complex roots.

Comment: The two solutions of this equation are complex numbers which are conjugates of each other. Notice that the solutions are written in standard a + bi format. The graph of $y = 4x^2 - 6x + 3$ does **not** cross the x-axis since the equation $4x^2 - 6x + 3 = 0$ has **no real** number solutions. Since the leading coefficient of the equation is positive, the parabola opens upward and therefore the graph of the parabola is above the x-axis. Refer to the properties of the discriminant above. The x-coordinate of the minimum point of the parabola equals ¾ which equals **-b/(2a)**.

Solve for a Variable in an Equation (Rearrange the Equation)

In each example below, one of the basic equation solving strategies is used to solve for the indicated variable.

(Convert Celsius to Fahrenheit)

Ex. 1 $12x - 6\mathbf{y} = 18$ (for **y**)

y
* ⁻6
+ 12x

(Rethink how to write the equation.)

$12x + (^-6y) = 18$

$-6y = 18 - 12x$

$$y = \frac{18 - 12x}{-6}$$

$y = -3 + 2x$

$y = 2x - 3$

Ex. 2 $2\mathbf{x} + 6y = 9$ (for **x**)

x
*2
+ 6y

$2x = 9 - 6y$

$$x = \frac{9 - 6y}{2}$$

Ex. 3 $F = \frac{9\mathbf{C}}{5} + 32$ (for **C**)

C
*9
/ 5
+ 32

$$F - 32 = \frac{9C}{5}$$

$5(F - 32) = 9C$

$$\frac{5(F - 32)}{9} = C$$

(Equation of an ellipse)

Ex. 4 $4x^2 + 27y^2 = 64$ (for y)

y
y^2
* 27
+ $4x^2$

$27y^2 = 64 - 4x^2$

$y^2 = \frac{64 - 4x^2}{27}$

$y = \pm\sqrt{\frac{64-4x^2}{27}}$

(Since **y** could be positive or negative, it **is necessary** to include $\pm$ symbol with the square root operation.)

(Equation for the surface area of a sphere.)

Ex. 5 $A = 4\pi r^2$ (for r)

r
r^2
$*4\pi$

$\frac{A}{4\pi} = r^2$

$r = \sqrt{\frac{A}{4\pi}}$

(Since **r** is always positive, it is **not** necessary to include $\pm$ symbol with the square root operation.)

(Equation for the volume of a sphere)

Ex. 6 $V = \frac{4\pi r^3}{3}$ (for r)

r
r^3
$*4\pi$
/ 3

$3V = 4\pi r^3$

$\frac{3V}{4\pi} = r^3$

$r = \sqrt[3]{\frac{3V}{4\pi}}$

(Never include a $\pm$ symbol when reversing the cube of a number operation.)

Ex. 7 P + Prt = 5,000 (for P)

(Get **P** by itself by factoring out **P**.)

P(1 + rt) = 5,000

P = $\frac{5{,}000}{1 + rt}$

Comment: Make sure that you completely understand the difference between examples **7** and **10**.

Ex. 8 $\frac{a-2}{b} = \frac{a+1}{c}$ (for **a**)

$ac - 2c = ab + b$

(Gather **a** on the same side of the equation.)

$ac - ab - 2c = b$

(Move **-2c** to the other side of the equation..)

$ac - ab = 2c + b$ (The variable **a** is now gathered on the left side of the equation..)

(Get **a** by itself by factoring out **a**.)

a(c – b) = 2c + b

a = $\frac{2c + b}{c - b}$

Ex. 9 $\frac{2}{a} - \frac{3}{b} = \frac{1}{c}$ (for **b**)

$abc\left(\frac{2}{a} - \frac{3}{b}\right) = abc\left(\frac{1}{c}\right)$ (Remove fractions by multiplying both sides by the lowest common denominator.)

$2bc - 3ac = ab$

$2bc - ab = 3ac$ (Gather the variable **b** on the left side of the equation..)

$b(2c - a) = 3ac$ (Factor out **b** .)

$b = \frac{3ac}{2c - a}$

Ex. 10. P + Prt = 5,000 (for **r**)

r
* Pt
+ P

Prt = 5,000 – P

r = $\frac{5{,}000 - P}{Pt}$

Ex. 11. $\frac{3y}{5} - \frac{y}{x+2} = 10$ (for y)

$5(x+2)\left(\frac{3y}{5} - \frac{y}{x+2}\right) = 5(x+2)10$

$3y(x+2) - 5y = 50(x+2)$

$3xy + 6y - 5y = 50x + 100$

$3xy + y = 50x + 100$

(The variable **y** is now gathered on the left side of the equation..)

$y(3x+1) = 50x + 100$

$y = \frac{50x + 100}{3x + 1}$

Ex. 12. Consider the **implicitly** defined relation $\mathbf{2x^2 - 3xy = 4y - 2}$. The graph of this relation is shown below. We will rewrite the relation as a function of **x** and then as <u>two functions</u> of **y**. The graphs of all of the equations are identical!

Rewrite as a function of x by solving for y.

$2x^2 + 2 = 3xy + 4y$ Gather **y** on the right side of equation.

$2x^2 + 2 = y(3x + 4)$ Factor out common factor of **y**.

$$y = \frac{2x^2 + 2}{3x + 4}$$

Observations:

The above formula expresses **y** as a function of **x**.
The domain of the function equals all real numbers except **x** = -4/3. The estimated range is $(y \le -4.0)$ **or** $(y \ge 0.4)$.

The numerator is always positive. Therefore the function has no real roots and the graph of the function can not intersect the **x**-axis.

When **x = -4/3**, the denominator equals zero. Therefore the line **x = -4/3** is a vertical asymptote of the graph.. As **x** gets closer to **-4/3** the function values approach plus or minus infinity.

When **x** = -4/3 + .0001, **y** = 18,516.74.
When **x** = -4/3 - .0001, y = -18,520.30

Rewrite as a function of y by solving for x.

Since there is a $\mathbf{x^2}$ and **x** in the equation, we will use the quadratic formula to solve for **x.**

$2x^2 - 3xy - 4y + 2 = 0$ Set equation equal to zero.

a = 2, b = -3y, and c = -4y + 2 Identify a, b, and c.

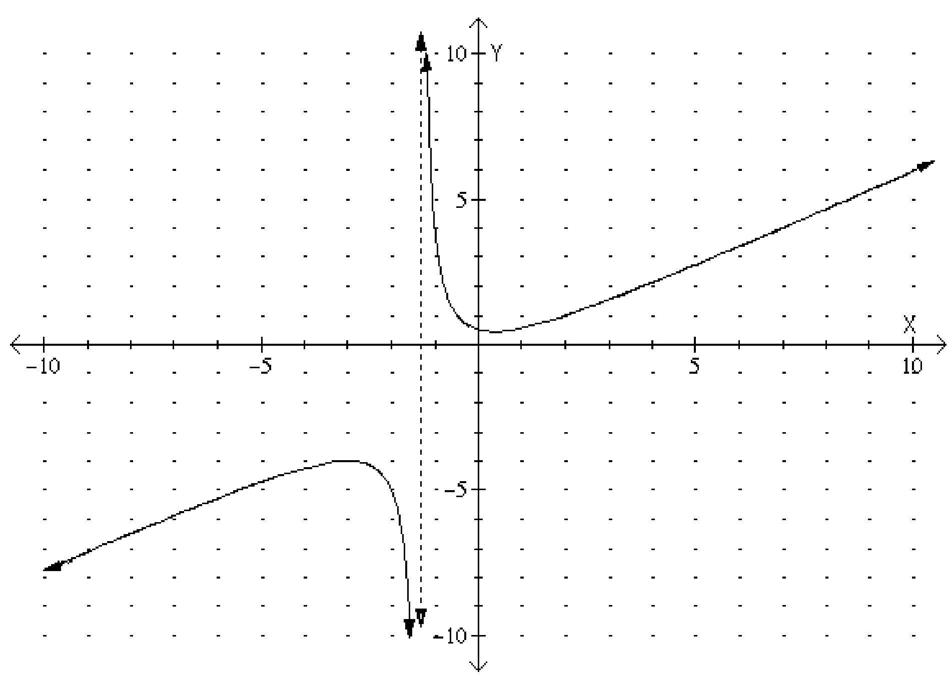
$$x = \frac{3y \pm \sqrt{(-3y)^2 - 4(-8y+4)}}{4}$$ Plug a, b, and c into the quadratic formula.

$$x = \frac{3y \pm \sqrt{9y^2 + 32y - 16}}{4}$$ Simplify expression.

Observations:

Because of the ± operator, the above formula expresses **x** in terms of **y**, but **not** as a single <u>function</u> of **y.** An input value for **y** will generally have two outputs. Recall that if a relation is also a function, then every in input value to the function can have no more than one output value. The estimated domain of this relation is $(y \le -4.0)$ **or** $(y \ge 0.4)$.

How can anyone not be impressed by the power of the quadratic formula!



These equations can be easily solved by applying simple properties of fractions, factoring and old fashioned thinking!

1. $\frac{x+3}{x-1}=\frac{4}{x-1}$

2. $\frac{5}{a}-\frac{4}{a}=8+\frac{1}{a}$

3. $\frac{x}{x-5}-\frac{5}{x-5}=3$

4. $\frac{a^2}{a+2}-\frac{4}{a+2}=a$

5. $\frac{z^2}{z+1}+2=\frac{1}{z+1}$

6. A mathematical proof that 1 = 2.
Can you find the mistake?

Let **a** and **b** be any two real numbers that are equal.

$a = b$	(So we let **a** equal **b**.)
$a^2 = ab$	(Multiply both sides by **a**)
$a^2 - b^2 = ab - b^2$	(Subtract $\mathbf{b^2}$ from both sides.)
$(a + b)(a - b) = b(a - b)$	(Factor both sides)
$\frac{(a+b)(a-b)}{(a-b)}=\frac{b(a-b)}{(a-b)}$	(Divide both sides by **a-b**)
$a + b = b$	(Simplify both sides.)
$\mathbf{b} + b = b$	(Substitute **b** for **a**)
$2b = b$	(Simplify left side)
$\frac{2b}{b}=\frac{b}{b}$	(Divide both sides by **b**.)
$\mathbf{2 = 1}$	(Simplify both sides of the quation.)